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Can Adults Teach Number Concepts to Young Children?

by Constance Kamii

When kindergarten teachers teach children how to count objects, many of them believe that they are teaching number concepts. However, children often count some objects more than once and overlook the others. Teachers usually correct these children, but on the next day the children go back to their incorrect ways.

To explain why it is so hard to teach kindergartners how to count objects, it is necessary to clarify the three kinds of knowledge Piaget (1945/1951; 1967/1971) distinguished according to their ultimate sources — physical knowledge, social-conventional knowledge, and logico-mathematical knowledge.

Physical knowledge is knowledge of objects in the external world. Knowing that marbles roll but chips do not is an example of physical knowledge. The fact that paper can easily be torn but cloth cannot is another example. The ultimate source of physical knowledge is objects in the external world.

An example of **social-conventional knowledge** is knowledge of languages like English and Spanish. Understanding the meaning of holidays like Thanksgiving is another example. Using rules of etiquette is also an example

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of social-conventional knowledge, which is found in the conventions people make over time.

Physical and social-conventional knowledge have sources outside the individual, but **logico-mathematical knowledge** is made, or constructed, by each individual. If I put a red chip and a white one in front of you and asked if the two chips are *different*, you would probably agree that the chips are *different*. If I asked you whether or not the same two chips are *similar*, you would also probably say "Yes." And if I asked you if there are *two* objects on the table, you would also agree that there are *two*. The same two chips can be *different, similar,* or *two* because these are all mental relationships we make by *thinking* about them. When we think about them as being *different*, they become different for us at that moment, and when we think about them as being *similar*, they become similar for us at that moment. And if we think about the chips as being *two*, they become *two* for us at that moment. Logico-mathematical knowledge thus has its source in each individual's thinking.

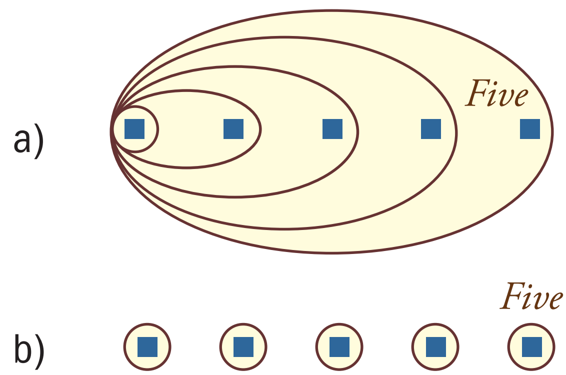
Another way of clarifying the nature of logico-mathematical knowledge is to say that the *difference* between the red and white chips is not observable because it (the difference) does not have an existence in the external world. Likewise, the *similarity* between the two chips does not have an existence in the external world, and the *two-ness* does not have an existence either. It is only when *we think about the two chips* as being *two* that the two chips become *two*.

If *two* is logico-mathematical knowledge that is made, or constructed, by each individual, all the other numbers like *three, four, five . . . ten, twenty, thirty . . . a hundred, five hundred,* and *a thousand* are also logico-mathematical knowledge that must be constructed by each individual.

This article is entitled "Can Adults Teach Number Concepts to Young Children?" The reader must have concluded by now that adults cannot teach number concepts to young children because children must construct, or create, them in their minds. It is possible, up to a point, to teach children to count objects because saying number words involves social-conventional knowledge. However, children's counting the same objects more than once, and skipping some, is usually impossible to correct for the reason elaborated by Piaget and Szeminska (1941/1965). These authors pointed out that the construction of number involves the synthesis of two kinds of logico-mathematical relationships: hierarchical inclusion and order.

Hierarchical Inclusion and Order

Figure 1. Counting Five Chips (a) with Hierarchical Inclusion and (b) without Hierarchical Inclusion



Hierarchical inclusion refers to the fact that when we (adults) count the five objects in Figure 1(a), we mentally include 'one' in 'two,' 'two' in 'three,' 'three' in 'four,' and 'four' in 'five.' But when kindergartners count these objects, and we ask them to 'show five,' many of them point to the last one saying "It's this one," as can be seen in Figure 1(b). This behavior indicates that the child has not constructed hierarchical inclusion. For him, saying "one-two-three" is like saying "Monday-Tuesday-Wednesday." Just as 'Monday' stands for one day, and 'Tuesday' for another day, 'one' refers to one object, and 'two' refers to another object.

Figure 2. Counting Five Chips with the Logico-Mathematical Relationship of Order

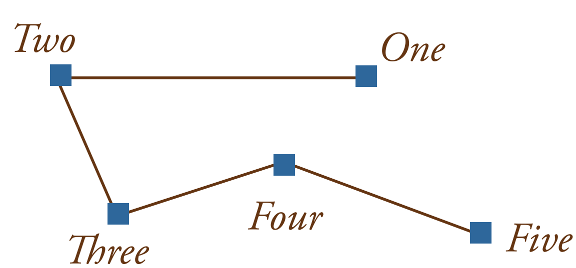


Figure 2 shows that when we (adults) count five randomly arranged objects, we put them in a **relationship of order** and know which one(s) have been counted and which one(s) remain to be counted. When kindergartners count the same objects, however, many of them count the same objects more than once and overlook the others as we have already seen. This behavior tells us that the child has not constructed the logico-mathematical relationship of order.

How Number Concepts can be Fostered

Number concepts cannot be taught *directly* as I have already stated, but there are many things adults can do *indirectly* to support children's construction of logico-mathematical knowledge. Examples of what adults can do can be found in daily living, such as when children spill milk, in activities like Pick-Up Sticks, and in games like Lining Up the 5s. These activities all encourage children to *think*, and to construct many aspects of logico-mathematical knowledge.

Situations in daily living. When a child spills milk, some teachers say, "You need to get a sponge and wipe it up," but others may ask, "What do you need to do?" The second teacher is encouraging the child to *think* much more than the first one. In the second situation, the child has to *think* and decide whether to get a mop, a sponge, or a paper towel.

The entire class gets disrupted all day long if students can sharpen their pencils at any time of the day. In this situation, one teacher may announce that everybody has to sharpen their pencils before 9:00. This way of solving the problem does not encourage children to think. Another teacher may say to the class, "We get disrupted every time somebody sharpens their pencil. Can anybody think of a way to solve this problem?" The rule children suggest may not work, but a rule can always be changed later.

Physical-knowledge activities. A physical-knowledge activity like **Pick-Up Sticks** is one in which children act on objects to produce a desired effect. In **Pick-Up Sticks**, children act on a stick trying not to move any other stick. The many kinds of logico-mathematical relationships children can make in this game can be seen in Figures 3 and 4.

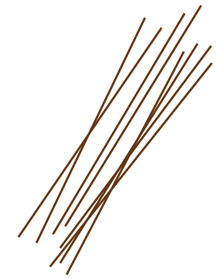


Figure 3. Eight sticks that have been scattered

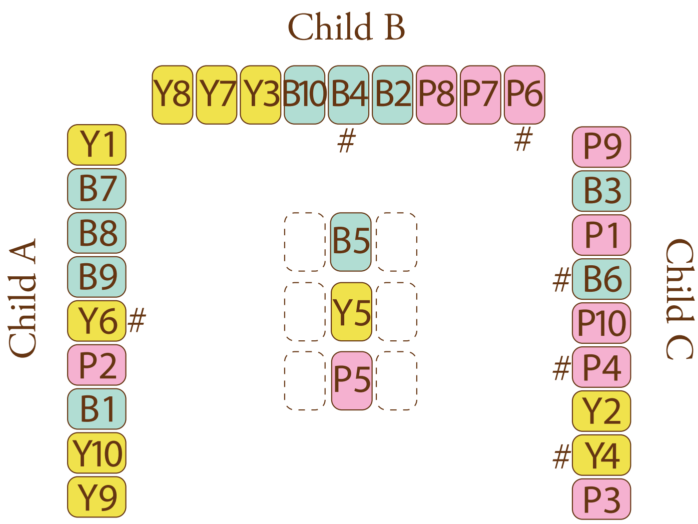
In Figure 3, the best sticks to try to pick up first are the two that are not touching any other stick. This is an example of *classification* in which the child classifies 'the sticks that are not touching any other stick' and 'those that are touching others.' There is thus a checkmark in Figure 4 under *Classification*. There is also a checkmark under *Seriation* because the child may then focus on the two sticks on the left-hand side. These are the *next hardest* to try to pick up, and the child continues to *seriate* the sticks when she goes on to the sticks that are even harder to try to pick up.

There are checkmarks under *Spatial* and *Temporal relationships* in Figure 4 because when the child has to deal with a stick that is on top of another, she simultaneously makes spatial and temporal relationships. To encourage children to *think*, we must be careful not to provide more than 8 or 10 sticks.

Figure 4. The Logico-Mathematical Relationships made in Cleaning Up, Pick-Up Sticks, and Lining Up the 5s

			Cleaning Up	Pick-Up Sticks	Lining Up the 5s
Physical knowledge					
Social-conventional					
Logico-mathmatical knowledge	Logico-arithmetic relationships	Classify relationships	■	■	■
		Seriational relationships	■	■	■
		Numerical relationships	■	■	■
	Spatio-temporal relationships	Spatial relationships	■	■	■
		Temporal relationships	■	■	■

Figure 5. Three children's arrangement of cards in Lining Up the 5s



Lining Up the 5s, a card game (Kato, Honda, & Kamii, 2006). This game uses 30 cards consisting of 10 cards each of numbers 1-10 in three different colors (available for downloading from my website, www.Constancekamii.org, in the category of "Materials for the Classroom"). The rules for this three-player game are the following:

All the cards are dealt, and the children align their cards face up as shown in Figure 5.

The players who have 5s put them down in a column in the middle of the table. ('B5' means 'Blue 5'; 'Y5' means 'Yellow 5'; and 'P5' means 'Pink 5.')

The group decides who will go first.

The players take turns putting down one card at a time. They make a matrix by extending each line, by color, to the right or left without skipping any number (6, 7, 8 . . . and 4, 3, 2, 1).

- Anyone who does not have a card to play must pass.

- The winner is the first person to use up all his cards.

It can be seen in Figure 5 that Child B was the only one who *classified* his cards according to color, and *seriated* them numerically within each category. The other two children aligned their cards haphazardly in ways that did not facilitate strategic planning.

In the situation in Figure 5, Child B could play his B4 or P6, as indicated by the '#' signs. He played his B4, which was a poor choice. If he played P6, he could subsequently use his P7 and P8 at any time, but he had to wait for Child C to play his B3 to be able to use his B2.

This game, too, encourages children to make all the logico-mathematical relationships in Figure 4. The players needed to *classify* their cards and everybody else's cards to know which card could be played, and *seriate* them to plan strategies. Child B was logically the most advanced child, but he did not make the *spatial* and *temporal relationships* necessary to notice the desirability of using P6 before B4.

In Conclusion

Adults cannot teach number concepts to young children directly in isolation, but can teach them indirectly by encouraging them to think throughout the day. As we saw in the framework in Figure 4, number is only one specific kind of logico-mathematical relationship. It is desirable for teachers to encourage children to make all kinds of logico-mathematical relationships because numerical relationships develop out of this complex network when children think.

References

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